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Radiation of Sound Waves from a Semi-Infinite Rigid Duct by Local Outer Lining

*[1]Burhan Tiryakioglu and [2]Ahmet Demir
*[1]Faculty of Arts and Sciences, Department of Applied Mathematics, Marmara University, Turkey
[2]Faculty of Engineering, Department of Mechatronics Engineering, Karabuk University, Turkey

Abstract

Radiation of sound waves from a semi-infinite rigid duct by local outer lining is investigated rigorously by the Wiener-Hopf technique. Through the application of Fourier-transform technique in conjunction with the Mode-Matching method, the radiation problem is described by a modified Wiener-Hopf equation of the third kind and then solved approximately. The solution involves a set of infinitely many expansion coefficients satisfying an infinite system of linear algebraic equations. Numerical solution of this system is obtained for various values of the parameters of the problem and their effects on the radiation phenomenon are presented.

Key words: Wiener-Hopf technique, Fourier transform, diffraction, duct

1. Introduction

As is well known, the propagation of sound in cylindrical ducts is a major problem in noise pollution. Accordingly, there have been a number of studies on radiation and propagation in ducts.

By using the Wiener-Hopf technique which is the efficient method for diffraction/radiation problems, the radiation of sound from a semi-infinite rigid pipe has been considered by Levine and Schwinger [1]. One of the effective methods of reducing unwanted noise which has been found empirically to be very successful is to create large additional sound absorption by lining the duct with an acoustically absorbent material. Rawlins who showed the effectiveness of this method, considered the radiation of sound from an unflanged rigid cylindrical duct with an acoustically absorbing internal surface [2]. Demir and Buyukaksoy solved same problem with partial linig [3]. They analyzed the effects of the length of internal surface impedance, pipe radius etc. with graphically for various values of the parameters.

In the present work, a rigorous approximate solution for the problem of sound waves from a semi-infinite rigid duct by local outer lining, is presented. This is similar to that considered by Demir and Buyukaksoy [3] which consists of partial internal impedance loading by a semi-infinite circular cylindrical pipe. In our work, as in the above study, hybrid method was applied.

By stating the total field in duct region in terms of normal waveguide modes (Dini's series) and using the Fourier Transform elsewhere, the related boundary value problem is formulated as a Modified Wiener-Hopf equation of the third kind whose formal solution involves branch-cut integrals with unknown integrands and infinitely many unknown expansion coefficients satisfying an infinite system of linear algebraic equations. Some computational results illustrating

*Corresponding author: Address: Faculty of Arts and Sciences, Department of Applied Mathematics, Marmara University, 34722, Istanbul, TURKEY. E-mail address: burhan.tiryakioglu@marmara.edu.tr, Phone:+902163451186

the effects of external surface impedance, duct radius etc. on the radiation phenomenon are presented.

A time factor $e^{-i\omega t}$ with ω being the angular frequency is assumed and suppressed throughout the paper.

2. Analysis

2.1. Formulation of the Problem

The geometry of the problem is sketched in Fig. 1. This geometry consists of a semi-infinite rigid duct with partial external lining. Duct walls are assumed to be infinitely thin, and they defined by $\{\rho = a, z \in (-\infty, l)\}$. The part $\rho = a, z \in (0, l)$ of its outer surface is assumed to be treated by an acoustically absorbent lining which is denoted by β, while the other parts of the duct are assumed to be rigid.

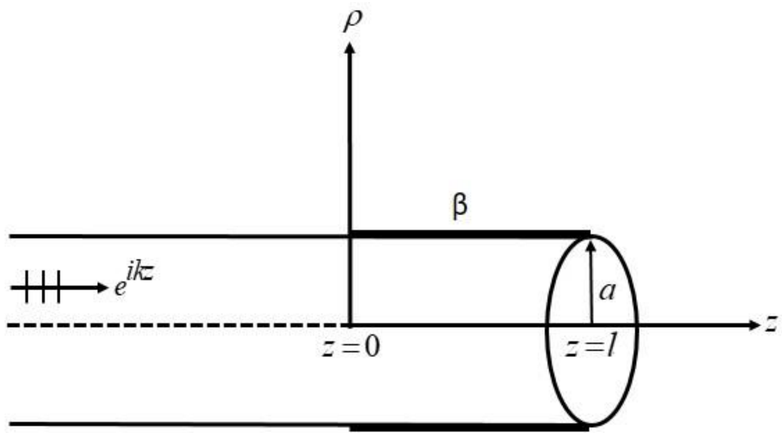

Figure 1. Geometry of the problem

From the symmetry of the geometry of the problem and of the incident field, the total field everywhere will be independent of θ.

The incident sound wave propagating along the positive z direction and is defined by

$$u^i = e^{ikz} \tag{1}$$

where $k = \omega/c$ denotes the wave number. For the sake of analytical convenience, the total field can be written in different regions as:

$$u^T(\rho, z) = \begin{cases} u_1(\rho, z) & , \ \rho > a \quad , \ z \in (-\infty, \infty) \\ u_2(\rho, z) & , \ \rho < a \quad , \ z \in (l, \infty) \\ u_3(\rho, z) + u^i & , \ \rho < a \quad , \ z \in (-\infty, l) \end{cases} \tag{2}$$

where u^i is the incident field as given by (1) and the fields $u_j(\rho, z), j = 1,2,3$ which satisfy the Helmholtz equation,

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial z^2}+k^2\right]u_j(\rho,z)=0 \quad , \quad j=1,2,3 \tag{3}$$

is to be determined with the help of the following boundary and continuity relations:

$$\frac{\partial}{\partial\rho}u_1(a,z)=0 \ , \ z<0 \quad , \quad \frac{\partial}{\partial\rho}u_3(a,z)=0 \ , \ z<l \tag{4a,b}$$

$$\left(ik\beta+\frac{\partial}{\partial\rho}\right)u_1(a,z)=0 \ , \ 0<z<l \tag{5}$$

$$\frac{\partial}{\partial\rho}u_1(a,z)-\frac{\partial}{\partial\rho}u_2(a,z)=0 \ ,z>l \quad , \quad u_1(a,z)-u_2(a,z)=0 \ ,z>l \tag{6a,b}$$

$$\frac{\partial}{\partial z}u_2(\rho,l)-\frac{\partial}{\partial z}u_3(\rho,l)=ike^{ikl}\ ,\rho<a \quad , \quad u_2(\rho,l)-u_3(\rho,l)=e^{ikl}\ ,\rho<a \tag{7a,b}$$

2.2. Derivation of the Modified Wiener-Hopf Equation

For $\rho>a, u_1(\rho,z)$ satisfies the Helmholtz equation for $z\in(-\infty,\infty)$. Multiplying (3) by $e^{i\alpha z}$ with α being the Fourier transform variable and integrating the resultant equation with respect to z from $-\infty$ to ∞, we obtain

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial z^2}+k^2\right]F(\rho,\alpha)=0 \quad , \quad z\in(-\infty,\infty) \tag{8}$$

with

$$F(\rho,\alpha)=\int_{-\infty}^{\infty}u_1(\rho,z)e^{i\alpha z}\,dz=F^-(\rho,\alpha)+F_1(\rho,\alpha)+e^{i\alpha l}F^+(\rho,\alpha) \tag{9}$$

where

$$F^-(\rho,\alpha)=\int_{-\infty}^{0}u_1(\rho,z)e^{i\alpha z}\,dz \quad , \quad F^+(\rho,\alpha)=\int_{l}^{\infty}u_1(\rho,z)e^{i\alpha(z-l)}\,dz \tag{10a,b}$$

$$F_1(\rho,\alpha)=\int_{0}^{l}u_1(\rho,z)e^{i\alpha z}\,dz \tag{10c}$$

here $F^+(\rho,\alpha)$ and $F^-(\rho,\alpha)$ are analytic functions on upper region $Im\alpha>Im(-k)$ and on lower region $Im\alpha<Imk$ of the complex α-plane, respectively, while $F_1(\rho,\alpha)$ is an entire function.

The general solution of (8) satisfying the radiation condition for $\rho>a$ reads

$$F^{-}(\rho,\alpha)+F_1(\rho,\alpha)+e^{i\alpha l}F^{+}(\rho,\alpha)=A(\alpha)H_0^{(1)}(K\rho) \tag{11}$$

where

$$K=\sqrt{k^2-\alpha^2} \quad , \quad K(0)=k \tag{12}$$

is the square root function (see Fig. 2) and

$$H_n^{(1)}(z)=J_n(z)+iY_n(z) \tag{13}$$

is the Hankel function of the first kind and n-th order. $A(\alpha)$ in (11) is a spectral coefficient to be determined.

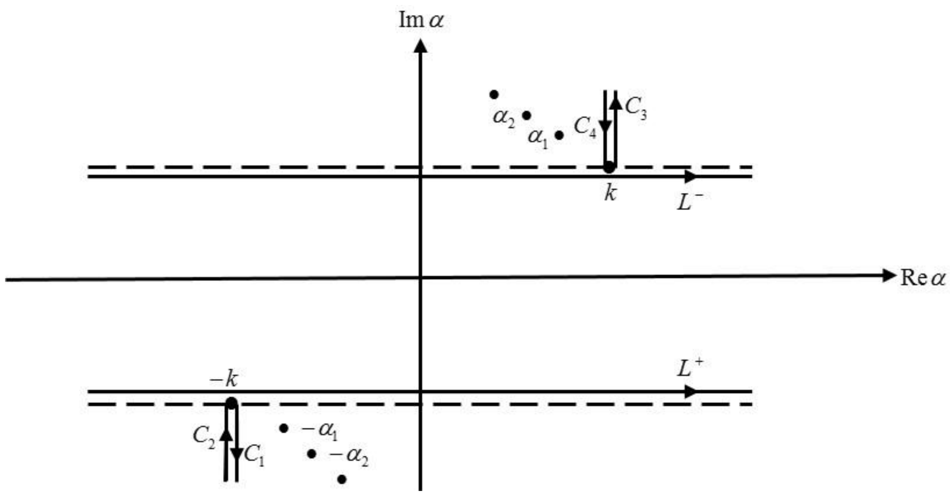

Figure 2. Branch-cut and integration lines in the complex plane

Consider now the Fourier transform of (4a) and (5), namely

$$\dot{F}^{-}(a,\alpha)=0 \quad , \quad ik\beta F_1(a,\alpha)+\dot{F}_1(a,\alpha)=0 \tag{14a, b}$$

where the dot specifies the derivative with respect to ρ. The differentiation of (11) with respect to ρ yields

$$\dot{F}^{-}(\rho,\alpha)+\dot{F}_1(\rho,\alpha)+e^{i\alpha l}\,\dot{F}^{+}(\rho,\alpha)=-KA(\alpha)H_1^{(1)}(K\rho) \tag{15}$$

Setting $\rho=a$ in (15) and using (14a,b), we obtain

$$A(\alpha)=\frac{W^{-}(\alpha)+e^{i\alpha l}W^{+}(\alpha)}{H(\alpha)} \tag{16}$$

where

$$W^{\pm}(\alpha)=ik\beta F^{\pm}(a,\alpha)+\dot{F}^{\pm}(a,\alpha) \tag{17}$$

$$H(\alpha)=ik\beta H_0^{(1)}(Ka)-KH_1^{(1)}(Ka) \tag{18}$$

The substitution of $A(\alpha)$ given in (16) into (11) yields

$$F^{-}(\rho,\alpha)+F_1(\rho,\alpha)+e^{i\alpha l}F^{+}(\rho,\alpha)=\left[W^{-}(\alpha)+e^{i\alpha l}W^{+}(\alpha)\right]\frac{H_0^{(1)}(K\rho)}{H(\alpha)} \tag{19}$$

In the region $\rho<a, z>l$. By taking the half-range Fourier transform of the Helmholtz equation satisfies by $u_2(\rho,z)$ we get

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{\partial^2}{\partial \mathrm{z}^2}+\mathrm{K}^2(\alpha)\right]G^{+}(\rho,\alpha)=f(\rho)-i\alpha g(\rho) \tag{20}$$

where

$$G^{+}(\rho,\alpha)=\int_l^\infty u_2(\rho,z)e^{i\alpha(z-l)}\,dz \tag{21}$$

while $f(\rho)$ and $g(\rho)$ stand for

$$f(\rho)=\frac{\partial}{\partial \mathrm{z}}u_2(\rho,l) \quad , \quad g(\rho)=u_2(\rho,l) \tag{22a, b}$$

$G^{+}(\rho,\alpha)$ is regular function in the upper $\big(Im\alpha>Im(-k)\big)$ half of the complex α-plane. The particular solution of (20) which is bounded and satisfying the impedance boundary condition at $\rho=a$ can be obtained by the Green's function technique. The Green's function related to (20) satisfies the Helmholtz equation.

$$\left[\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\mathrm{K}^2(\alpha)\right]\acute{\mathrm{G}}(\rho,t,\alpha)=0 \quad , \quad \rho\neq t \quad , \quad \rho,t\in(0,a) \tag{23}$$

the solution is

$$\acute{\mathrm{G}}(\rho,t,\alpha)=\frac{1}{J(\alpha)}Q(\rho,t,\alpha) \tag{24a}$$

with

$$Q(\rho,t,\alpha)=\frac{\pi}{2}\begin{cases}J_0(K\rho)[J(\alpha)Y_0(Kt)-Y(\alpha)J_0(Kt)] & , \ 0\leq\rho\leq t\\ J_0(Kt)[J(\alpha)Y_0(K\rho)-Y(\alpha)J_0(K\rho)] & , \ t\leq\rho\leq a\end{cases} \tag{24b}$$

where

$$J(\alpha)=ik\beta J_0(Ka)-KJ_1(Ka) \tag{24c}$$
$$Y(\alpha)=ik\beta Y_0(Ka)-KY_1(Ka) \tag{24d}$$

The solution of (20) can now be written as

$$G^{+}(\rho,\alpha)=\frac{1}{J(\alpha)}\left[B(\alpha)J_0(K\rho)+\int_0^a(f(t)-i\alpha g(t))Q(t,\rho,\alpha)\,tdt\right] \tag{25}$$

In (25), $B(\alpha)$ is a spectral coefficient to be determined, while f and g are given by (22a,b). Combining (6a) and (6b) and using Fourier transform, we can write

$$ik\beta G^{+}(a,\alpha) + \dot{G}^{+}(a,\alpha) = W^{+}(\alpha) \tag{26}$$

Substituting (25) and its derivative with respect to ρ in (26), one can obtain $B(\alpha) = W^{+}(\alpha)$ Inserting now $B(\alpha)$ into (25), we get

$$G^{+}(\rho,\alpha) = \frac{1}{J(\alpha)}\left[W^{+}(\alpha)J_0(K\rho) + \int_0^a (f(t) - i\alpha g(t))Q(t,\rho,\alpha)\,tdt\right] \tag{27}$$

The left hand side of (27) is regular in the upper half plane. The regularity of the right hand side may be violated by the presence of simple poles occurring at the zeros of $J(\alpha)$ lying in the upper half plane, namely at $\alpha = \alpha_m$, $m = 1,2,\ldots$

$$ika\beta J_0(\gamma_m) - \gamma_m J_1(\gamma_m) = 0 \quad , \quad \alpha_m = \sqrt{k^2 - \left(\frac{\gamma_m}{a}\right)^2} \quad , \quad Im(\alpha_m) \geq Imk \tag{28}$$

We can eliminate these poles by imposing that their residues are zero. This gives

$$W^{+}(\alpha_m) = \frac{a}{2}J_0(\gamma_m)[1 - (\beta ka/\gamma_m)^2][f_m - i\alpha_m g_m] \tag{29}$$

with

$$\begin{bmatrix} f_m \\ g_m \end{bmatrix} = \frac{2}{a^2 J_0^2(\gamma_m)[1 - (\beta ka/\gamma_m)^2]}\int_0^a \begin{bmatrix} f(t) \\ g(t) \end{bmatrix} J_0\left(\frac{\gamma_m}{a}t\right) tdt \tag{30a}$$

Owing to (30a), $f(\rho)$ and $g(\rho)$ can be expanded into Dini series as follows;

$$\begin{bmatrix} f(\rho) \\ g(\rho) \end{bmatrix} = \sum_{m=1}^{\infty} \begin{bmatrix} f_m \\ g_m \end{bmatrix} J_0\left(\frac{\gamma_m}{a}\rho\right) \tag{30b}$$

Using the continuity relation (6b) and combining (19) with (27), we get the following Modified Wiener-Hopf Equation (MWHE) of the third kind.

$$\frac{a}{2}F^{-}(a,\alpha)N(\alpha) + \frac{e^{i\alpha l}W^{+}(\alpha)}{M(\alpha)} - \frac{a}{2}F_1(a,\alpha) = e^{i\alpha l}\frac{a}{2}\sum_{m=1}^{\infty}\frac{J_0(\gamma_m)}{\alpha_m^2 - \alpha^2}[f_m - i\alpha g_m] \tag{31a}$$

where

$$M(\alpha) = \pi i J(\alpha)H(\alpha) \quad , \quad N(\alpha) = \frac{KH_1^{(1)}(Ka)}{H(\alpha)} \tag{31b,c}$$

2.3. Approximate Solution of the Modified Wiener-Hopf Equation

By using the factorization and decomposition procedures, together with the Liouville theorem, the modified Wiener-Hopf equation in (31a) can be reduced to the following system of Fredholm integral equations of the second kind:

$$\frac{W^+(\alpha)}{M_+(\alpha)} = -\frac{1}{2\pi i}\frac{a}{2}\int_{L_+}\frac{M_-(\tau)N(\tau)e^{-i\tau l}F^-(a,\tau)}{(\tau-\alpha)}d\tau + \frac{1}{2\pi i}\frac{a}{2}\int_{L_+}\frac{M_-(\tau)}{(\tau-\alpha)}\sum_{m=1}^{\infty}\frac{J_0(\gamma_m)}{\alpha_m^2-\tau^2}[f_m - i\tau g_m]d\tau \qquad (32a)$$

$$\frac{a}{2}F^-(a,\alpha)N_-(\alpha) = \frac{1}{2\pi i}\int_{L_-}\frac{e^{i\tau l}W^+(\tau)}{N_+(\tau)M(\tau)(\tau-\alpha)}d\tau - \frac{1}{2\pi i}\frac{a}{2}\int_{L_-}\frac{e^{i\tau l}}{N_+(\tau)(\tau-\alpha)}\sum_{m=1}^{\infty}\frac{J_0(\gamma_m)}{\alpha_m^2-\tau^2}[f_m - i\tau g_m]d\tau \qquad (32b)$$

where $M_+(\alpha),\ N_+(\alpha)$ and $M_-(\alpha),\ N_-(\alpha)$ are the split functions, analytic and free of zeros in the upper and lower halves of the complex $\alpha -$ plane, respectively, resulting from the Wiener-Hopf factorization of $M(\alpha)$ and $N(\alpha)$ which are given by (31b) and (31c), in the following form:

$$M(\alpha) = M_+(\alpha)M_-(\alpha) \quad , \quad N(\alpha) = N_+(\alpha)N_-(\alpha) \qquad (33a,b)$$

Here the explicit forms for $M_+(\alpha),\ M_-(\alpha)$ and $N_+(\alpha),\ N_-(\alpha)$ can be obtained as is done in [4]. For large argument, the coupled system of Fredholm integral equations of the second kind in (32a,b), is susceptible to a treatment by iterations. Now, the approximate solution of the MWHE reads:

$$\frac{W^+(\alpha)}{M_+(\alpha)} \approx \frac{a}{2}\sum_{m=1}^{\infty}\frac{J_0(\gamma_m)[f_m + i\alpha_m g_m]M_+(\alpha_m)}{2\alpha_m(\alpha+\alpha_m)} + \frac{a}{2}\sum_{m=1}^{\infty}\frac{J_0(\gamma_m)[f_m - i\alpha_m g_m]e^{i\alpha_m l}}{2\alpha_m N_+(\alpha_m)}I_1(\alpha) \qquad (34a)$$

$$F^-(a,\alpha)N_-(\alpha) \approx -\sum_{m=1}^{\infty}\frac{J_0(\gamma_m)[f_m - i\alpha_m g_m]e^{i\alpha_m l}}{2\alpha_m N_+(\alpha_m)(\alpha-\alpha_m)} + \sum_{m=1}^{\infty}\frac{J_0(\gamma_m)[f_m + i\alpha_m g_m]M_+(\alpha_m)}{2\alpha_m}I_2(\alpha) \qquad (34b)$$

with

$$I_1(\alpha) = \frac{1}{2\pi i}\int_{L_+}\frac{M_-(\tau)N(\tau)e^{-i\tau l}}{(\tau-a)(\tau-\alpha_m)N_-(\tau)}d\tau \ , \ I_2(\alpha) = \frac{1}{2\pi i}\int_{L_-}\frac{M_+(\tau)e^{i\tau l}}{N_+(\tau)M(\tau)(\tau-a)(\tau+\alpha_m)}d\tau \qquad (35a,b)$$

The integral given by (35a,b) can be obtained numerically. First the integration line $L_{\pm}$ can be deformed onto the branch-cut, then the integrals can be evaluated by the help of Cauchy theorem.

2.4. Reduction to an Infinite System of Linear Algebraic Equations

In the region $\rho < a, z < l,\ u_3(\rho, z)$ can be expressed as

$$u_3(\rho, z) = \sum_{n=0}^{\infty} c_n e^{-i\sigma_n z} J_0\left(\frac{J_n}{a}\rho\right) \tag{36a}$$

with

$$J_1(J_n) = 0 \quad , \quad \sigma_n = \sqrt{k^2 - \left(\frac{J_n}{a}\right)^2} \quad , \quad \sigma_0 = k \tag{36b}$$

From the continuity relations (7a,b) and using (30b).

$$\sum_{m=1}^{\infty} f_m J_0\left(\frac{\gamma_m}{a}\rho\right) = -\,i \sum_{n=0}^{\infty} \sigma_n c_n e^{-i\sigma_n l} J_0\left(\frac{J_n}{a}\rho\right) + ike^{ikl} \tag{37a}$$

$$\sum_{m=1}^{\infty} g_m J_0\left(\frac{\gamma_m}{a}\rho\right) = \sum_{n=0}^{\infty} c_n e^{-i\sigma_n l} J_0\left(\frac{J_n}{a}\rho\right) + e^{ikl} \tag{37b}$$

Then multiply both sides of (37a,b) by $\rho J_0\left(\frac{J_l}{a}\rho\right)$ and integrate from $\rho = 0$ to $\rho = a$. We get

$$\sum_{m=1}^{\infty} \frac{J_0(\gamma_m)}{\gamma_m^2}[f_m + ikg_m] = \frac{e^{ikl}}{a\beta} \quad , \quad n = 0 \tag{38a}$$

$$\frac{2ka\beta}{\sigma_n J_0(J_n)} \sum_{m=1}^{\infty} \frac{J_0(\gamma_m)}{\gamma_m^2 - J_n^2}[f_m + i\sigma_n g_m] = 0 \quad , \quad n = 1,2, \ldots \tag{38b}$$

By substituting $\alpha = \alpha_1, \alpha_2, \alpha_3, \ldots$ in (34a) and using (29) we obtain

$$\frac{J_0(\gamma_r)[1 - (\beta ka/\gamma_r)^2][f_r - i\alpha_r g_r]}{M_+(\alpha_r)} = \sum_{m=1}^{\infty} \frac{J_0(\gamma_m)[f_m + i\alpha_m g_m]M_+(\alpha_m)}{2\alpha_m(\alpha_r + \alpha_m)} + \sum_{m=1}^{\infty} \frac{J_0(\gamma_m)[f_m - i\alpha_m g_m]e^{i\alpha_m l}}{2\alpha_m N_+(\alpha_m)} I_1(\alpha_r) \tag{39}$$

(38a,b) and (39) are the required linear system of algebraic equations which permits us to determine f_m and g_m. The coefficient c_m can be found from f_m or g_m. All the numerical results will be derived by truncating the infinite series and the infinite systems of linear algebraic equations after first N term. It was seen that the amplitude of the radiated field becomes insensitive to the increase of the truncation after $N = 20$.

2.5. Analysis of the Field

The total field in the region $\rho > a$ can be obtained by taking the inverse Fourier transform of $F(\rho,\alpha)$. From (11) and (16), one can write

$$u_1(\rho,z) = \frac{1}{2\pi}\int_{\mathcal{L}} ik\beta F^-(a,\alpha)\frac{H_0^{(1)}(K\rho)}{H(\alpha)}e^{-i\alpha z}d\alpha + \frac{1}{2\pi}\int_{\mathcal{L}} W^+(\alpha)\frac{H_0^{(1)}(K\rho)}{H(\alpha)}e^{-i\alpha(z-l)}d\alpha \quad (40a)$$

where $\boldsymbol{\mathcal{L}}$ is a straight line parallel to the real α-axis, lying in the strip $Im(-k) < Im\alpha < Imk$. Utilizing the asymptotic expansion of Hankel function and using the saddle point technique, we get

$$u_1(\rho,z) \sim \frac{k}{i\pi}\left[\frac{ik\beta F^-(a,-kcos\theta_1)}{\mathrm{H}(-kcos\theta_1)}\frac{e^{ikr_1}}{kr_1} + \frac{W^+(-k\cos\theta_2)}{\mathrm{H}(-kcos\theta_2)}\frac{e^{ikr_2}}{kr_2}\right] \quad (40b)$$

where r_1, θ_1 and r_2, θ_2 are the spherical coordinates defined by

$$\rho = r_1 sin\theta_1 \quad , \quad \mathrm{z} = r_1 cos\theta_1 \quad (41a)$$

and

$$\rho = r_2 sin\theta_2 \quad , \quad \mathrm{z} - \mathrm{l} = r_2 cos\theta_2 \quad (41b)$$

3. Numerical Results

In this section, some graphics displaying the effects of the radius of the pipe, the external lining etc. on the radiation phenomenon are presented. The far field values are plotted at a distance 46 m away from the duct edge. Figure 3 shows the variation of the amplitude of the radiated field against the observation angle for different values of the impedance. From Figure 4, it is observed that the radiated field increases with increasing duct radius, as expected.

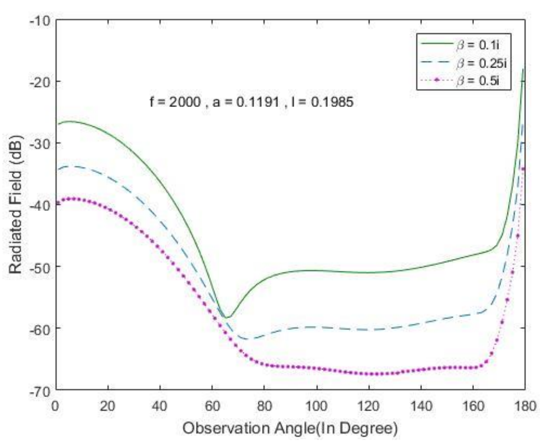

Figure 3. $20log|u_1|$ versus the observation angle for different values of the impedance.

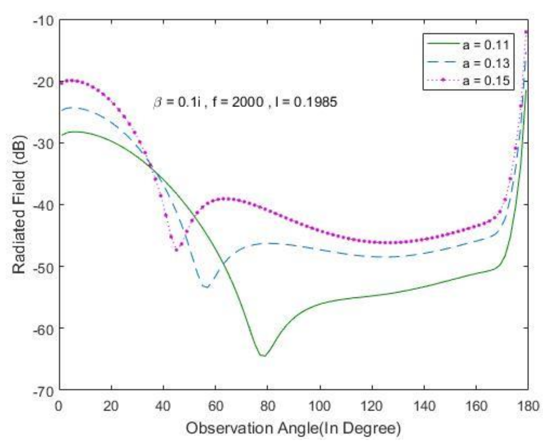

Figure 4. $20log|u_1|$ versus the observation angle for different values of the duct radius.

Figure 5 shows the variation the modules of reflection coefficient $|c_0|$ with respect to frequency f for different positive values of the lining impedance β. Finally Figure 6 displays the variation of the amplitude of the radiated field versus f.

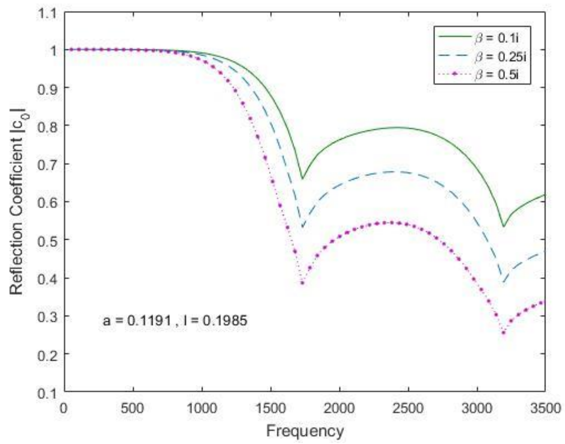

Figure 5. Variation of the reflection coefficient $|c_0|$ versus frequency f.

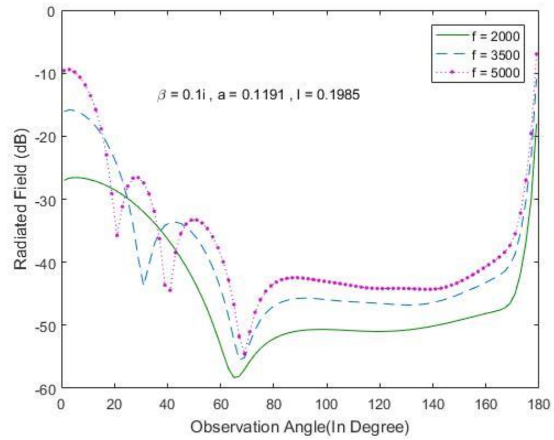

Figure 6. $20log|u_1|$ versus f.

4. Conclusions

The radiation of sound waves from a semi-infinite rigid duct whose outer surface is treated by an acoustically absorbing material of finite length, has been investigated by using the mode-matching method in conjunction with the Wiener-Hopf technique. This problem is more complicated due to partial lining of the external surface. To overcome the additional difficulty caused by the finite impedance discontinuity, the problem was first reduced to a system of Fredholm integral equations of the second kind and then solved approximately by iterations. The solution involves two systems of linear algebraic equations involving two sets of infinitely many unknown expansion coefficient. A numerical solution to these systems has obtained for various values of the parameters such as radius a, outer partial impedance β and wave number k.

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